## NC Programme of study

- Add numbers using concrete objects and pictorial representations including:
- A two digit number and ones
- A two digit number and tens
- Two ten digit numbers
- Adding three one digit numbers
- Show that addition of two numbers can be done in any order
- (Non Stat: record addition in columns supports place value and prepares for formal written methods)


## BY THE END OF YEAR 2.

Children will be confident in using a range of base 10 equipment in columns. They will be able to add two 2-digit numbers and start to record the value of the digits in the tens and ones columns.

## Using grouped objects for addition, without recombining

Once secure with the understanding of place value for two-digit numbers, teachers should demonstrate, using concrete objects grouped in tens, how numbers can be represented and partitioned and then used for calculating.

'Familiar' objects should be used initially. Pegs packaged into packs of ten are ideal, and they clearly show the number contained on the labelling. Discuss the contents and count the contents of a pack with the children. Discuss the value of the single pegs as 'ones'.

Show how a number such as 23 can be represented using the pegs. How many packs of ten pegs can we use? How many pegs will that be? How many single pegs will we need to make our total?

We also have 12 pegs. Write an addition calculation such as $23+$ 12. Ensure the ones do not add to more than 9 , and thus won't need to be recombined. How can we show the extra 12 pegs? (One pack of ten and 2 single pegs). Place the 23 pegs and the 12 pegs directly underneath each other.


Demonstrate how to combine the single pegs first, using the vocabulary of addition, bringing them to the bottom of the grid. board. Bring the packs of ten pegs down in the same way. Allow the children to determine how to calculate the total number of pegs, encouraging the use of 'tens and ones' vocabulary. Model the counting in tens for the packs of pegs, and in ones for
 the single pegs.
Once secure, teachers can annotate alongside the concrete objects. This starts to link to the next stage in the progression towards a formal written method, where calculations are laid out vertically.
Try to integrate the inverse operation of subtraction as much as possible. How many would we have left if we subtracted 12 from our total? How do you know? What if we subtracted 23 ? Demonstrate this by moving the pegs back up the grid.

## Using grouped objects for addition, with recombining

Once children are secure with the concept of combining the grouped objects for numbers that do not require re-grouping (i.e. ones do not add to more than 9 ), then numbers can be chosen that will require re-grouping. Bundles of straws are the next step in the progression of being really secure with base-ten place value. They are easy to manipulate, yet allow the children to still see the 'ten-ness' of ten, allowing for simple regrouping.


Show ' $34+27$ '. Place the grouped concrete objects (moving on to straws bundled into tens) and discuss the value of the different groups in the same way as above.

Combine the 'ones' straws and move them to the bottom of the grid. Use mental calculation skills to find how many single straws there are altogether. What can we do now that we have 11 straws?
Explain the 'regrouping' of ten of the straws (using an elastic band) and then position these at the bottom of the tens column. What do we need to do now?

Bring the tens bundles down to the bottom of the grid. How many straws do we have altogether?

As before, the link with subtraction can be integrated here, discussing what would happen if 34 straws were subtracted from the total. What if we subtracted 27 ? Note that to demonstrate this, one of the straw bundles would need to be split (see decomposition for subtraction), and so a careful discussion would need to take place.


Children working confidently with bundles of straws can move on to using Dienes base-ten equipment. This is also grouped in tens, but cannot be split apart or recombined easily, and requires an understanding of exchange. The Dienes equipment should be introduced alongside the straws, enabling the children to see what is the same and what is different.


The next step is to record the calculation in books, thus: $34+22$
III : :

## NC Programme of study

- Subtract numbers using concrete objects and pictorial representations including:
- A two digit number and ones
- A two digit number and tens
- Two ten digit numbers
- Adding three one digit numbers
- Show that addition of two numbers can be done in any order
- (Non Stat: record subtraction in columns supports place value and prepares for formal written methods)


## BY THE END OF YEAR 2...

Children will be confident in using a range of base 10 equipment in columns. They will be able to subtract two 2-digit numbers and start to record the value of the digits in tens and ones columns.

## Using grouped objects for subtraction, without exchanging

Once secure with the understanding of place value for two-digit numbers, teachers should demonstrate, using concrete objects grouped in tens, how numbers can be represented on grids and then used for calculating.

'Familiar' objects should be used initially. Pegs packaged into packs of ten are ideal, and they clearly show the number contained on the labelling. Discuss the contents and count the contents of a pack with the children. Discuss the value of the single pegs as 'ones'.

Show how a number such as 28 can be represented using the pegs. How many packs of ten pegs can we use? How many pegs will that be? How many single pegs will we need to make our total?

Place the pegs on the grid.


We need to give away 5 pegs, so we will subtract them from the 28 . Write the subtraction calculation '28-5'. Ensure the units being subtracted do not exceed those in the initial number, i.e. 8 , and thus exchanging from tens to ones will not be necessary.

Demonstrate how to subtract the 5 single pegs from the 8 , by moving them down on the grid. Do we need to move any of these packs of 10 away? Why not? Ensure children understand that the ones digit is changing, but the tens digit is not.

Model how the remaining pegs are moved to the 'answer line' at the bottom of the grid. How many pegs do we have left? How many tens? How many ones?

Once secure, teachers can annotate alongside the concrete objects. This starts to link to the next stage in the progression towards a formal written method, where calculations are laid out vertically.


Recording: $28-5=$

$-\frac{20$| 8 |
| ---: |
| 5 |}{$20+3$}$=23$

Ensure the inverse operation of addition is integrated as much as possible. If l added my 5 pegs back on, how many pegs would I have? Model this by working backwards, up the grid, resulting in the same image the children started with.

## Using grouped objects for subtraction, with exchanging

Once children are secure with the concept of subtracting the grouped objects for numbers that do not require exchanging (i.e. the ones in the number being subtracted do not exceed those in the starting number), then numbers can be chosen that will require exchanging. Bundles of straws are the next step in the progression of being really secure with base-ten place value. They are easy to manipulate, yet allow the children to still see the 'ten-ness' of ten, allowing for simple regrouping.


Show the calculation ' $43-27$ '. Lay the 43 grouped concrete objects (moving on to straws bundled into tens) onto the grid and discuss the value of the different groups in the same way as above.

Discuss the fact that 7 ones or single straws need to be removed. Why is this difficult? Do we have enough straws? Discourage the children from saying that they 'can't do it', and explain that they need to split one of the groups of ten into ten separate straws. Demonstrate this by removing the elastic band from one bundle of ten, and move the ten separate straws to the 'ones' column. Emphasise that 43 is now 'thirty and thirteen', simply arranged in a different way.


Using the same action as previously, model how the 7 'ones' straws can be moved down the grid, followed by the 2 tens bundles. Explain each step carefully to the children.


Model how the remaining straws are moved to the 'answer line' at the bottom of the grid. How many straws do we have left? How many tens? How many ones? How many straws did we start with? How many did we take away?

Ensure the inverse operation of addition is integrated as much as possible. If / added my remaining 16 straws to the 27 I took away, how many would I have? Why? Model this by working backwards, up the grid. Show how, with regrouping of one bundle of ten straws, the result is the same image the children started with.

Recording:


As understanding develops, children should move onto using grouped base-ten equipment that requires an understanding of exchange, such as 'Dienes'. This new equipment should be introduced alongside the straws, enabling the children to see what is the same and what is different.


## NC Programme of study

- Recall and use multiplication and division facts for the 2,5,10 tables including recognising odd and even numbers
- Calculate mathematical statements for
multiplication and division within the multiplication tables and write them using symbols $\times$ / = / $\div$


## BY THE END OF YEAR 2...

Children should be able to recognise arrays in printed form and in the environment, and be able to describe them in terms of repeated addition as well as with a multiplication statement.

Following on from year 1...
Making arrays using physical objects and representing them in drawings


Children should be familiar with the array image for multiplication from their experiences in Year 1.
Children in year 2 should continue their experiences with arrays, arranging counters, drawing their own representations, counting and labelling the groups.
E.g. 2 groups of 4 (starting from the left of the array and counting rows) or 4 groups of 2 (starting from the left of the array and counting rows).

Show that multiplication of two numbers can be done in any order (commutative).

## Annotating arrays using repeated addition



Children should be encouraged to see the array as a number of counters repeated in rows.
E.g. $4+4+4=12$

$$
3+3+3+3=12
$$

N.B. These examples encourage children to see an array being built up from the top row, which links well to later work for division. Arrays are viewed as being built up from top to bottom.

Annotating arrays using multiplication

$4+4+4+4+4=20 \quad 5 \times 4=20$
This annotation links to the idea of multiplication as 'scaling'; making a number so many times bigger. Here the starting number is 4 and it is 'scaled up' five times, or by a factor of five.

Children should record as shown:


## Recognising arrays in the environment

Children should be encouraged to recognise arrays in the local environment and use these to support their multiplication skills. Examples could include wrapping or wallpaper patterns, flowers and other natural objects, cinema seats, bricks on a wall, windows or balconies on a building.


Using jottings to develop an understanding of doubling
Towards written methods:

Ask the children to use jottings to develop an understanding of doubling two digit numbers.

The example here shows 16 doubled, using partitioning:


Investigate the appearance of odd numbers in an array.
Which numbers cannot be arranged complete rows?
(1, 2, 3, 5, 7, 11, 13, 17...)

NC Programme of study

- Recall and use division facts for the 2,5,10 tables including recognising odd and even numbers
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using symbols $x /=/ \div$
- Solve problems involving multiplication and division using materials, arrays, repeated addition, mental methods, multiplication and division facts


## BY THE END OF YEAR 2.

By the end of Year 2, children will be able to show their understanding as:


When the children can discuss their division confidently, using the language of both sharing and grouping accurately, the mathematical signs should be introduced for accompanying number sentences, e.g. $8 * 4=2$ and $8 * 2=4$

When dividing by 2, use this as an opportunity to link with fractions work.
E.g. $8 \div 2$... How can we read this? How many groups of 2 are there in 8 ? If I share 8 between 2 people, how many would they each get? What is one half of 2 ?

Links should also be made to multiplication work, e.g. $2 \times 4=8,4 \times 2=8$, modelling practically using arrays.

Remind the children that multiplication of two numbers can be done in any order and division of one number by another cannot.

Considering remainders when dividing


## Using base-ten resources to support division

As the numbers increase in size, base-ten grouped practical resources should replace separate single objects.


Ask the children how 12 can be represented with the base-ten (Dienes) equipment. Discuss the need to divide the 12 by 2 . Shall we share the 12 between 2 , or shall we group the 12 into 2 s?

Welcome their ideas for 'splitting' the 12. If necessary, suggest exchanging the single 10 rod for 10 separate ones. Carry out the practical dividing action, showing the link between sharing and grouping once the array is formed.

Extension: When the children are secure, physically dividing the baseten resources and forming an array, provide examples where remainders occur, e.g. $12 \div 5$

Again model how the single 10 rod is exchanged for ten separate ones. These should then be carefully arranged into an array, showing the groups of 2 , with the remaining 2 ones. Model the recording of this as $12 \div 5=2$ remainder $2(12 \div 5=2$ r 2$)$


